REFLECTIONS ON THE SACRED GEOMETRY OF THE TRIPLE TAU AND CIRCUMPUNCT

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INTRODUCTION

This investigation of the geometrical significance of the well-known Royal Arch Triple Tau symbol was inspired by a talk recently given by the High Priest of Fredericksburg Royal Arch Chapter No. 23, and District Education Officer for the 19th Capitular District, Excellent Richard Fisher, and is based on the research paper by Excellent H. Meij, *The Tau and the Triple Tau*, as retrieved from the website of the Grand Chapter of Royal Arch Masons of Virginia (Ref. 1).

Excellent Meij's paper connected the tau cross with the Hebrew word תָּו (*tav*), meaning mark, as used in the following passage of Scripture—

> *And the LORD said unto him, Go through the midst of the city, through the midst of Jerusalem, and set a* <u>*mark*</u> *upon the foreheads of the men that sign and that cry for all the abominations that be done in the midst thereof.* [Ezekiel 9:4]

According to Genesius' Hebrew-Chaldee lexicon (Ref. 2), תָּו (Strong's H8420) signifies a brand on horses or camels, or a cross made in lieu of a signature, *e.g.*, for those who were illiterate. Further, this mark would exempt the bearer from Divine judgement—

> *And to the others he said in mine hearing, Go ye after him through the city, and smite: let not your eye spare, neither have yet pity:*
>
> *Slay utterly old and young, both maids, and little children, and women: but come not near any man upon whom is the* <u>*mark*</u>*, and begin at my sanctuary. Then they began at the ancient men which were before the house.* [Ezekiel 9:5—6]

Genesius further points out that in Phoenician, the letter *taw* used in branding said animals had the form of a four-armed cross, whence the Greeks and Romans took the name and form of their letters.

Excellent Meij then goes on to relate the Triple Tau to the British Companion's Jewel, which incorporates the Seal of Solomon, as well as to the five Platonic solids (*i.e.*, the tetrahedron, cube, octahedron, icosahedron, and dodecahedron) and their mathematical properties. This paper will explore further some of the geometrical attributes and Hermetic significance of this important symbol. We will also examine an interesting property of that significant First Degree symbol, the Circumpunct, or Point within a Circle.

GEOMETRY OF THE TRIPLE TAU

A common form of the triple Tau symbol, inscribed inside an equilateral triangle, which is itself circumscribed by a circle, is as follows:

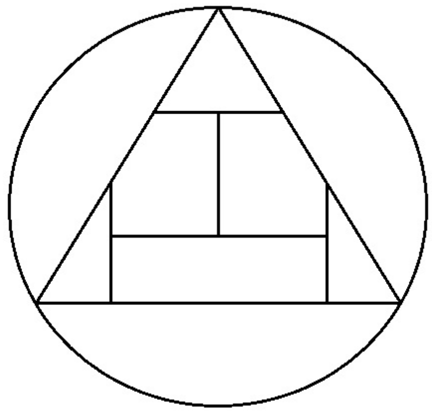


Figure 1

Several variations on this basic form are seen, including forms in which the arms of the crosses do not reach the sides of the triangle, forms where the arms are longer or shorter than the upright, forms with or without the circle, etc. There does not seem to be common agreement as to what the size or relative proportions of the constituent crosses should be. This is reminiscent of the situation regarding the various forms of the Square and Compasses in Symbolic Masonry, where the relative arrangement of these Lights, *e.g.*, angle to which the Compasses are opened, varies in different traditions. For the purpose of this paper, we will consider the geometric properties of a form such as that shown above—that is, where the arms are extended to touch the sides of the triangle, which is an equilateral triangle inscribed in a circle.

However, this is not enough to uniquely determine the proportions of the figure. Let us therefore stipulate the following constraint, which results in a special case for which we will draw certain philosophically meaningful conclusions—

The three tau crosses have the same size and shape (i.e., are congruent).

The reason for this is that the Triple Tau is often considered to be a symbol of the Trinity, and as such, represents the three Persons of the Godhead, co-equal, and balanced in perfect harmony. It thus stands to reason that, to preserve this symbolism, the three crosses should be equal in size and shape.

The equilateral triangle is also recognized as symbol of Deity, and so we might insist that the upper triangle should, like the larger triangle, be equilateral. But this is automatically true as seen from the following figure:

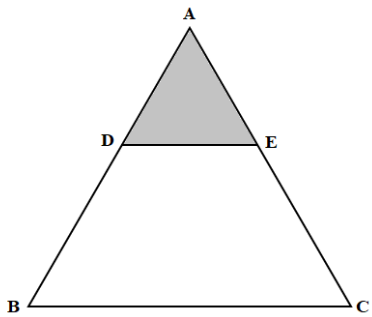


Figure 2

The segment *DE* is parallel to the base of the triangle *BC*, so the small, shaded triangle *ADE* is similar to the large triangle *ABC*. This can easily be seen by observing that the interior angles are all the same in both triangles.

Thus, if the large triangle is equilateral, so is the small triangle (with interior angles all being 60°).

The uppermost triangle is therefore a symbol of Deity, at the pinnacle of existence, just as the tau's are a symbol of the Trinity. We might further suppose the whole triangle to represent the Macrocosm and the upper triangle, smaller but of identical form, to represent the Microcosm.

With the above condition, it may be intuitive to some that with the three tau's being congruent, the ends of the tau's must touch, such that their crosspieces form the sides of a perfect square, itself leading to an important Hermetic symbol. A detailed geometric proof of this is given in the following section. This section can be skipped for those who find this sort of thing tedious. However, geometry being the foundation on which the superstructure of Masonry is founded, this has been included to demonstrate that the application of logic—being the third of the Liberal Arts and Sciences—to certain basic postulates, such as our condition above, can lead to surprising and interesting results and open the door to new truths. This might be considered illustrative of the way basic moral principles are gradually unfolded and built upon the progression of the Mysteries. Through the application of logic, precept upon precept, we gradually build the three-dimensional form of our Temple.

DETAILED GEOMETRIC CONSTRUCTION AND PROOF

We now proceed to construct Fig. 3, which is the main figure used throughout the remainder of this study. For simplicity, we'll assume the sides of the large triangle are equal to 1. This causes no loss of generality because it is only the relative proportions we're interested in. The construction of the figure and proof that congruence of the tau crosses leads to a square inscribed within the triangle is as follows—

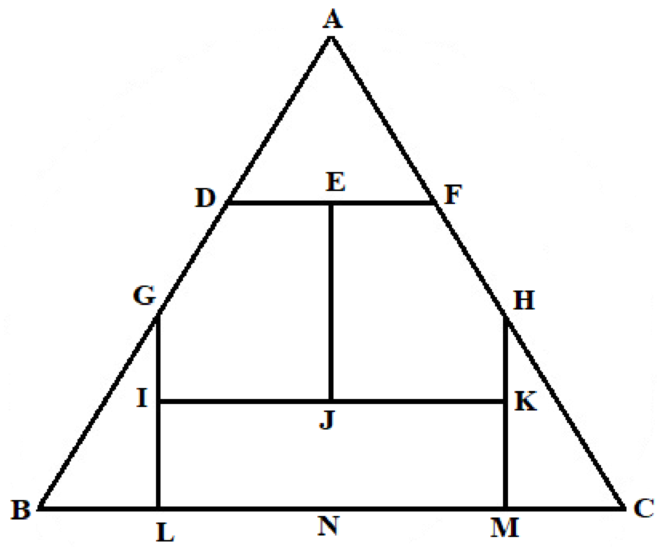


Figure 3

1. Construct an equilateral triangle *ABC*, with length of each side set equal to 1.

2. Construct a horizontal line segment from Point *D* to Point *F* and define $a = |DF|$ (where $|DF|$ is the length of the line segment). Let *E* be the midpoint of segment *DF*, such that $|DE| = |EF| = a/2$.

3. Construct vertical line segments *GL* and *HM*, such that $|GL| = |HM| = a$. Let Points *I* and *J* be the midpoints of *GL* and HM respectively, such that $|GI| = |IL| = |HK| = |KM| = a/2$.

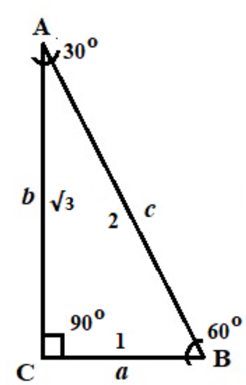
4. Construct the horizontal line segment *IK* and define $b = ½|IK|$. Let Point *J* be its midpoint, such that $|IJ| = |JK| = b$.

5. Because *IK* is parallel to *LM*, and *IK* and *LM* are perpendicular to *GL* and *HM*, $|LM| = |IK| = 2b$. We can therefore write $|BC| = |BL| + |LM| + |MC|$, or $1 = |BL| + 2b + |MC|$.

   Triangle *BGL* is a 30-60-90 right triangle, where $|GL| = a$. By the properties of a 30-60-90 right triangle, as shown in Figure 4, we know that $|BL| = a/\sqrt{3}$. [This step makes use of the Pythagorean Theorem, also known as the 47th Problem of Euclid.] Similarly, $|MC| = a/\sqrt{3}$.

   Substituting these values into the equation above, we can write this as $1 = 2a/\sqrt{3} + 2b$. Rearranging, we obtain

$$b = \frac{1}{2} - \frac{a}{\sqrt{3}}$$



$$a^2 + b^2 = c^2$$

$$1^2 + (\sqrt{3})^2 = 2^2$$

Figure 4

6. Construct the vertical line segment *EJ* and define $c = |EJ|$.

7. As was done in Step 5 for the base of triangle *ABC*, the height $|AN| = |AE| + |EJ| + |JN|$. Since triangle *ANC* is a 30-60-90 right triangle of base $|NC| = ½$, by Figure 4, $|AN| = \sqrt{3}/2$.

   Similarly, since triangle *AEF* is a 30-60-90 right triangle of base $|EF| = a/2$, $|AE| = a\sqrt{3}/2$. [Note that a is the factor by which triangle *AEF* is smaller than triangle *ANC*.]

   Because segments *IK* and *LM* are parallel (both horizontal), the perpendicular distance between them is constant and therefore $|JN| = |IL| = |KM| = a/2$.

   Therefore, substituting these values into the equation for the height of triangle *ABC* at the beginning of this step, $\sqrt{3}/2 = a\sqrt{3}/2 + c + a/2$. Rearranging, we obtain

$$c = \frac{\sqrt{3}}{2} - a\frac{1+\sqrt{3}}{2}$$

8. We now return to the special case in which the three tau's are congruent, in which case $b = c$. Setting the expression for b in Step 5 equal to that for c in Step 7, we obtain

$$\frac{1}{2} - \frac{a}{\sqrt{3}} = \frac{\sqrt{3}}{2} - a\frac{1+\sqrt{3}}{2}$$

   Collecting the constant terms on the left-hand side and terms in a on the right, we can solve for a:

$$a = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) \Big/ \left(\frac{1}{\sqrt{3}} - \frac{1+\sqrt{3}}{2}\right)$$

   We first write $1/\sqrt{3}$ as $\sqrt{3}/3$, then multiply by 2/2 to simplify to $(1-\sqrt{3})/(2\sqrt{3}/3 - (1+\sqrt{3}))$. Then multiply by $\sqrt{3}/\sqrt{3}$, then reverse signs in the numerator and denominator, simplifying to $a = (3-\sqrt{3})/(1+\sqrt{3})$, which can be further simpified as follows:

$$a = \frac{3-\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{3 - 3\sqrt{3} - \sqrt{3} + 3}{1-3} = \frac{6-4\sqrt{3}}{-2} = 2\sqrt{3} - 3$$

9. Substituting the value of a in Step 8 into the above expression for b, we can show that

$$b = \frac{1}{2} - \frac{(2\sqrt{3}-3)}{\sqrt{3}} = \frac{1}{2} - 2 + \frac{3}{\sqrt{3}} = \sqrt{3} - \frac{3}{2} = a/2$$

10. Substituting the value of a in Step 8 into the above expression for c, we can show that

$$c = \frac{\sqrt{3}}{2} - (2\sqrt{3} - 3)\frac{1 + \sqrt{3}}{2} = \frac{1}{2}\{\sqrt{3} - (2\sqrt{3} - 3)(1 + \sqrt{3})\} = \frac{1}{2}\{\sqrt{3} - (3 - \sqrt{3})\} = \frac{1}{2}(2\sqrt{3} - 3)$$
$$= \sqrt{3} - \frac{3}{2} = a/2$$

Thus, for the special case in which the three tau's are congruent, we get the final simple result that $a = 2\sqrt{3} - 3$; moreover, $b = c = a/2$. We sought to prove that this special case—where the tau's are congruent and reach the sides of the triangle—exists (though we have not demonstrated that this solution is unique) and have determined proportions a, b, and c which satisfy this condition. We will now see that we can easily confirm what we suspected above, that the ends of the tau's touch and form a perfect square.

GEOMETRIC AND SPIRITUAL SIGNIFICANCE OF THE TRIPLE TAU

After all of this mathematics, we find that the length of the crosspiece is approximately 46% (*i.e.*, a factor of $a = 2\sqrt{3} - 3$) of the length of one side of the large triangle and twice the length of the upright. But the length of the crosspiece is equal to $2a$. This means that all three arms of each of the tau crosses are of equal length!

This is an interesting result, because it means that if we assume the three tau's are the same shape and size, and allow the crosses to reach the sides of the triangle, we get three crosses each made up of three equal segments. That is, there are 9 equal lengths in all! This is shown in Fig. 5 below, though we have drawn the tau's with more typical proportions (for now). Because all 9 segments are equal, they can be rearranged into 3 equilateral triangles, as also shown in Fig. 5.

Recall that both the triangles and tau's are symbols of the Trinity. This fact is suggestive of the idea that, not only are the three Persons of the Trinity co-equal to one another, but each Person of the Trinity contains the full Nature of the Godhead. Each Person of the Trinity—Father, Son, and Holy Ghost—individually contains the fullness of the Deity. In the traditional doctrine of the Trinity, there is but One God, Three in One. Each member of the Godhead is not one third of God, but fully God.

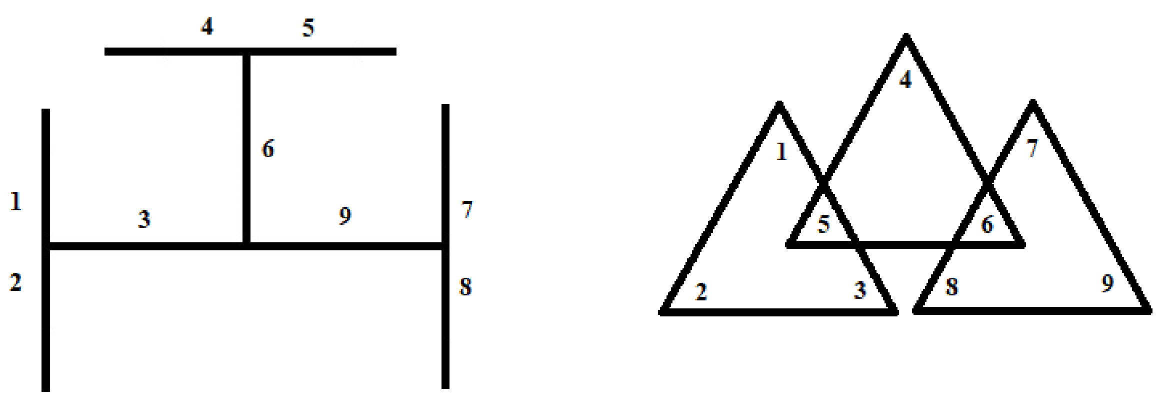


Figure 5

It would be improper to write down all the ways this teaching is echoed in the Mysteries, but a Royal Arch Mason can follow this thread with a little reflection. Mathematically, this means that God must be infinite, because it would be nonsensical to hold that 1/3 of a finite thing can be equal to the whole. However, those familiar with Georg Cantor's theory of transfinite cardinals knows that any infinite subset of a set of cardinality $\aleph_0$ has the same cardinality as the original set: $\aleph_0 = (1/3)\aleph_0$.[1] It is interesting that we began with the *tav* and end with the *aleph*, the beginning and end of the Hebrew alphabet (AΩ).

This is also reminiscent of the *sephiroth* of Kabbalah, which are Divine attributes or emanations, each of which contains, as does each of the four worlds, all the others within it[2] (Ref. 3). Both the Triple Tau and the *sephiroth* exhibit this idea of *self-similarity*—each part is a miniature representation of the whole. [Yet another topic this reminds us of is that of the well-known *golden ratio*, occurring in both nature and architecture, $\varphi = ½(1 + \sqrt{5})$. Interestingly, this involves the square root of 5 (or $\sqrt{5}$), while the Triple Tau involves the square root of 3 (or $\sqrt{3}$), square roots of two of the sacred numbers of Freemasonry. But those are subjects for other papers.]

Looking to re-draw Figure 3 with its correct proportions, we notice that the 3 crosses must actually touch on the perimeter of the triangle. That is, on the right-hand side, Points *F* and *H* are the same vertical distance $b = a/2$ from the base of the triangle. That means that if we start with tau crosses with a longer upright than crosspiece (as it is often pictured), and then shrink the upright, we will slide down the right-hand side of the triangle until *F* and *H* converge. The same is true of Points *D* and *G* on the left-hand side. We will leave it to an exercise to show that $FH = GD = 0$. Therefore, the true form is shown below on the left side of Figure 6.

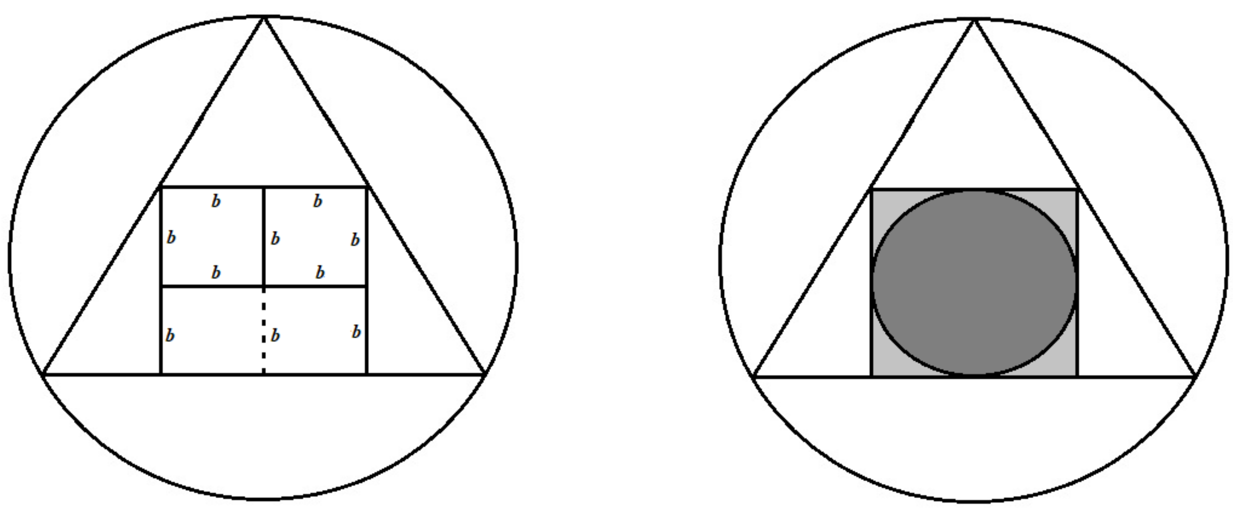


Figure 6

[1] $\aleph_0$, pronounced '*aleph-null*,' represents the size of the set of all whole numbers. The number of whole numbers is, paradoxically, equal to the number of even numbers, odd numbers, multiples of 3, multiples of 4, etc. (see also Hilbert's Hotel Paradox).

[2] From *Morals and Dogma*, 28° Lecture, p. 863 (also illustrated graphically in the plates on p. 906 – 907).

This means that the shape subtended by the three tau crosses is a square! The sides of the square are of size $2b = a$ (remembering that the sides of the triangle have been set to 1), a square of area $a^2 = 4b^2$. If it is a square, we can inscribe a circle of radius b inside it. We do this with a compass—thus, we see both the square and compasses coming into play. The center of the circle is the point at which the three tau's join, and the circle touches the square where the crosspieces intersect the uprights. Considering only the top and bottom, or two sides, of this square, the result is the circle embordered by two parallel lines.

In addition, this is exactly the form of the Hermetic Seal of Light, an alchemical symbol consisting of an equilateral triangle inscribed in a circle, with a square inscribed in the triangle and another circle inscribed in the square. There are many rectangles that can be inscribed in an equilateral triangle, but only one square—the square we constructed whose side is roughly 46.4% as long as the side of the triangle. Additionally, the square divides the triangle into a top equilateral triangle and two 30-60-90 right triangles, which together make up another equilateral triangle, the same way two oblong rectangles can be put together to make a perfect square. (Note that these triangles, both being equilateral, are necessarily similar but are not the same size, *i.e.*, not congruent.) Thus, by imposing the condition that the three tau crosses representing the three Persons of the Trinity be of equal shape and size, we have produced the Hermetic Seal, in which the three shapes revered by the Pythagoreans represent mercury, sulfur, and salt—and are also said to represent the physical body, soul, and spirit, collectively representing Quintessence.[3] We leave you with an illustration from Michael Maier's alchemical work *Atalanta Fugiens* (Fig. XXI).

Inspection of this figure will reveal that the triangle enclosing the square need not be equilateral, as it was in our special case. There are many such isosceles triangles that could be constructed around the square.

[3]I haven't found a good book reference for this, other than some websites (e.g., Ref. 4).

Figure 7

GEOMETRY OF THE CIRCUMPUNCT

Euclid's *Elements* is one of the crowning achievements of human intellect in the ancient world—nay, in all of human history. Another example of the application of the sacred science of geometry, 'being of a Divine and moral nature,' to the moral lessons of Masonry is found in the *circumpunct*, or point within a circle, embordered by two perfect parallel lines, which is introduced in the First Degree of Masonry. Consider the figure below.

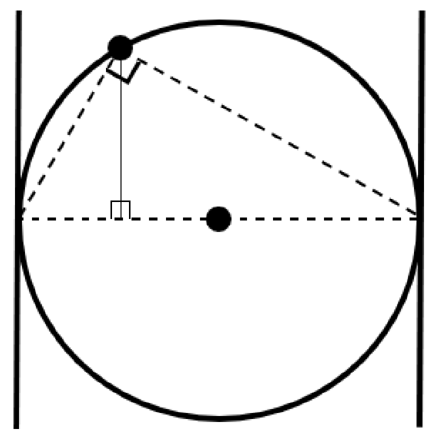


Figure 8

'In going round this Circle we necessarily touch upon those two Parallel Lines as well as the Book of Constitutions; and while a Mason keeps himself circumscribed within their precepts, it is impossible he can materially err.'

It is known that Albert Pike considered the Book of Constitutions to be a later affixation to this symbol,[4] so we set that aside here. Note that the Mason *goes round the circle*, which is contrary to interpretations that say the center represents the individual Mason[5], who cannot circle himself. Some hold the lines represent the Tropics of Cancer and Capricorn, bounding the path of the Sun's passage through the Heavens over the course of a year. Note the above section from the written work of the Entered Apprentice Lecture states that the two parallel lines are tangent to the circle. They cannot both be *tangent* and *parallel* unless they lie across the circle's diameter, *i.e.*, their points of contact lie at opposite ends of the diameter.

Book III, Proposition 31, of Euclid's *Elements*[6] states:

<u>In a circle the angle in the semicircle is right</u>, that in a greater segment less than a right angle, and that in a less segment greater than a right angle; and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.

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[4] *Morals and Dogma* (Ref. 3), 1° Lecture (p. 89).
[5] *Webb's Freemason's Monitor, Including the First Three Degrees, etc.*, Chapter VIII (1865).
[6] Translated by Sir Thomas Heath, Dover's Second Edition, unabridged (1956).

This is worded confusingly, but it is given here to convey the complexity of the ideas presented in the original Greek. The important part for us is the underlined portion, which is also known as Thales' Theorem. In modern English, it states simply that if we inscribe a triangle inside a circle—that is, all three points lie on the circumference of the *circumscribing triangle*—and the base of the triangle is a diameter of the circle (if it divides the circle into semicircles), then the opposite angle must be a right angle.

This can be proven from Proposition 8 of Book VI:

If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.

This means that if we drop a perpendicular to the hypotenuse of a right triangle, it divides the triangle into two triangles that are *similar* to each other and to the original triangle.[7] One proof entails the fact that the angles of any triangle sum to 180°, so the two angles other than the right angle must be *complementary*, meaning they sum to 90°. From this, it can be shown that the two angles on either side of the perpendicular are complementary to each other, and therefore taken together they form a right angle.

A consequence of Book III, Proposition 31 is that, in moving around this circle, if we draw a line from our location to each of these points of contact, they necessarily form a right angle. That is, *if in moving through the course of our lives* (and Masonic journeys), *we keep the Holy Saints John firmly in view* (they being our points of connection to, and immediate manifestation of, those abstract dual principles symbolized by the two columns), *we will maintain the righteous form—the due form—of a right angle, a Square of Virtue.* [The Holy Saints John represented the polarity of God's Judgement and His Mercy, being the Prophet of Wrath and the Apostle of Love.]

We won't go through this in detail, but Thales' Theorem represents an important method in the ancient world of constructing a right angle, just as Euclid's 47th Problem, or 47th Proposition (*i.e.*, Proposition 47 from Book I of his *Elements*), represents another. This is stated as:

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

And conversely, Book I, Proposition 48 is:

If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.

In modern language, the 48th Proposition states that if the sides A, B, and C of a triangle satisfy the Pythagorean Equation $A^2 + B^2 = C^2$, then the angle opposite side C is a right angle. This, and not the 47th Proposition, is what allows this test to be used to construct a right angle. The simplest test is to use the Pythagorean triplets, or groups of whole numbers that satisfy this condition, the smallest one being 3, 4, and 5. However, there are an infinite number of such triplets (*e.g.*, 3-4-5, 5-12-13, 8-15-17, *etc.*, along with their scalar multiples) that could be used. Much has been written about the mystical and spiritual significance of the 3-4-5 right triangle, less so on the intimately connected and nearly equally important 8th and 31st Propositions. Consider, however,

[7] *Similar* triangles have the same angles and their sides are in the same proportions.

that in going round the courses of our lives, we sometimes are closer to St. John the Baptist and sometimes closer to St. John the Evangelist. While one angle increases, the other decreases, and vice versa, yet taken in their proper balance, the sum of these complementary angles always forms a perfect right angle of virtue. As long as we keep ourselves circumscribed within their precepts, we will always be just and upright Masons.

REFERENCES

Does not include general references relied on, e.g., The Holy Bible (KJV) and Euclid's *Elements*.

1. H. Meij, *The Tau and the Triple Tau*, Tokyo Chapter No. 1, RAM (2000).
2. H. F. W. Genesius, *Hebrew and Chaldee Dictionary to the Old Testament Scriptures*, Transl. S. P. Tregelles, Samuel Bagster and Sons, London (1857).
3. Albert Pike, *Morals and Dogma of the Ancient and Accepted Scottish Rite of Freemasonry: Annotated Edition*, 2nd Ed., Supreme Council 33°, S.J., Washington, DC (2013).
4. *https://www.ancient-symbols.com/alchemy-symbols.html*, retrieved Nov. 23, 2022.
5. Michael Maier, *Atalanta Fugiens*, Johann-Theodor de Bry, Oppenheim, Germany (1617).